MEI Further Pure 1 Induction and Series

Section 1: Proof by Induction

Exercise level 3

1. Prove by induction that

$$2(2)+3(2^2)+4(2^3)+\cdots+(n+1)(2^n)=n(2^{n+1})$$

for all positive integers n.

Hence show that

$$1(2)+2(2^2)+3(2^3)+\cdots+98(2^{98})=97(2^{99})+2$$

2. The Fibonacci sequence $F_1, F_2, F_3, \ldots$ is defined by $F_1=1, F_2=1$ and $F_{n+1}=F_n+F_{n-1}$ $(n\geq 2)$.

(i) Write down the values of $F_3, F_4, \ldots, F_{10}$.

(ii) Prove that $F_{n+5}=5F_{n+1}+3F_n$.

(iii) Prove by induction that F_{5n} is a multiple of 5 for $n\geq 1$.

