MEI Further Pure 1 Induction and series

Section 1: Proof by induction

Exercise level 2

In Questions 1 to 7 prove the given result by induction.

1. $\sum_{r=1}^{n} r^2(r+1) = \frac{n(n+1)(n+2)(3n+1)}{12}$

2. $\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \frac{(n+2)}{2^n}$

3. $\sum_{r=1}^{n} 2\times 3^r = 3\left(3^n - 1\right)$

4. For a sequence defined by $u_{n+1} = 3u_n + 2$ and $u_1 = 1$ for $n \geq 1$, $u_n = 2\left(3^{n-1}\right) - 1$.

5. Given that $u_{(n+1)} = 2u_n + 1$ where n is a positive integer and $u_1 = 5$
   $u_n = 3 \times 2^n - 1$.

6. If $\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$, $\mathbf{A}^n = \begin{pmatrix} 2n+1 & -n \\ 4n & 1-2n \end{pmatrix}$ where n is a positive integer.

7. If $\mathbf{M} = \begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}$, $\mathbf{M}^n = \begin{pmatrix} 2^n & 0 \\ 2^{n+1}-2 & 1 \end{pmatrix}$ where $n \geq 1$.

Enrichment questions

8. Prove by induction that $n^3 + 3n^2 - 10n$ is a multiple of 3 for all positive integers n.

9. Prove by induction that $3^{2n} - 1$ is a multiple of 8 for all positive integers n.