Ritangle 18 – Questions 1-25 – Answers

1. Any such number must be divisible by 9 and by 2.

So it must be even and end in a 2.

The digits must also sum to a multiple of 9.

Seven 1s and a 2 add to 9, so 11111112 is one possible number.

Seven 3s and a 2 add to 23, so the only other possibility for the digit sum is 18.

We could arrive at this in three ways;

Four 3s, two 2s and two 1s, giving $\binom{7}{4}\binom{3}{2}$ possibilities.

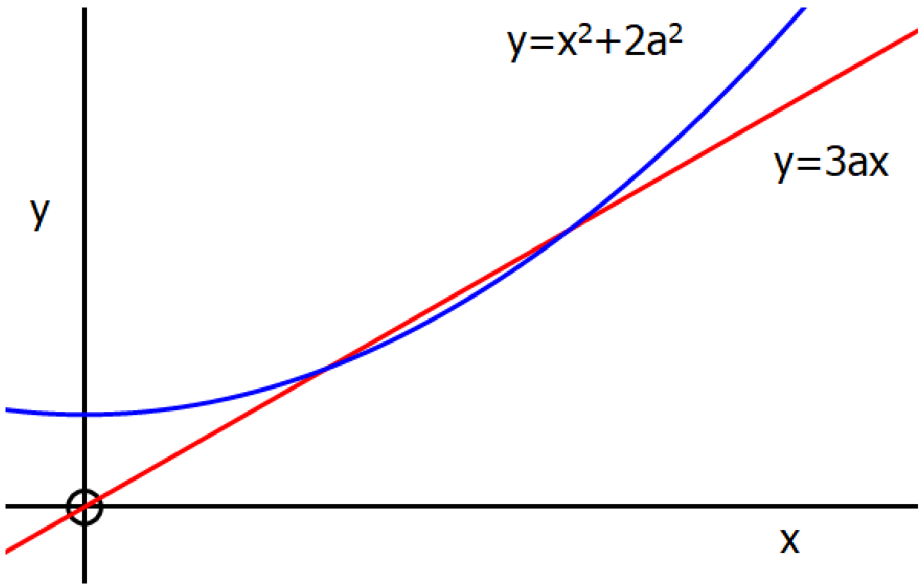
Three 3s, four 2s and a 1, giving $\binom{7}{3}\binom{4}{1}$ possibilities.

Two 3s and six 2s, giving $\binom{7}{2}$ possibilities.

Adding these up, we get 267 possible numbers.

So 267 is the final answer.

2.



Where do $y = x^2+2a^2$ and $y = 3ax$ cut? Solving $x^2+2a^2 = 3ax$ gives $x = a, 2a$. So

$$a = \int_a^{2a} 3ax\,dx - \int_a^{2a} x^2 + 2a^2\,dx = \left[\frac{3ax^2}{2} - \frac{x^3}{3} - 2a^2x\right]_a^{2a}$$

$$= \left(6a^3 - \frac{8a^3}{3} - 4a^3\right) - \left(\frac{3a^3}{2} - \frac{a^3}{3} - 2a^3\right) = \frac{a^3}{6}$$

Solving gives $a = \sqrt{6}$.

So on multiplying by 10^7 and taking the integer part, 24494897 is the final answer.

3. The remainder theorem gives us that

$10a^2 + 100a + 10 = b$, $10b^2 + 100b + 10 = a$.

Subtracting, we have $10(a^2 - b^2) + 100(a - b) = b - a$.

Dividing by a - b (since $a - b \neq 0$) gives

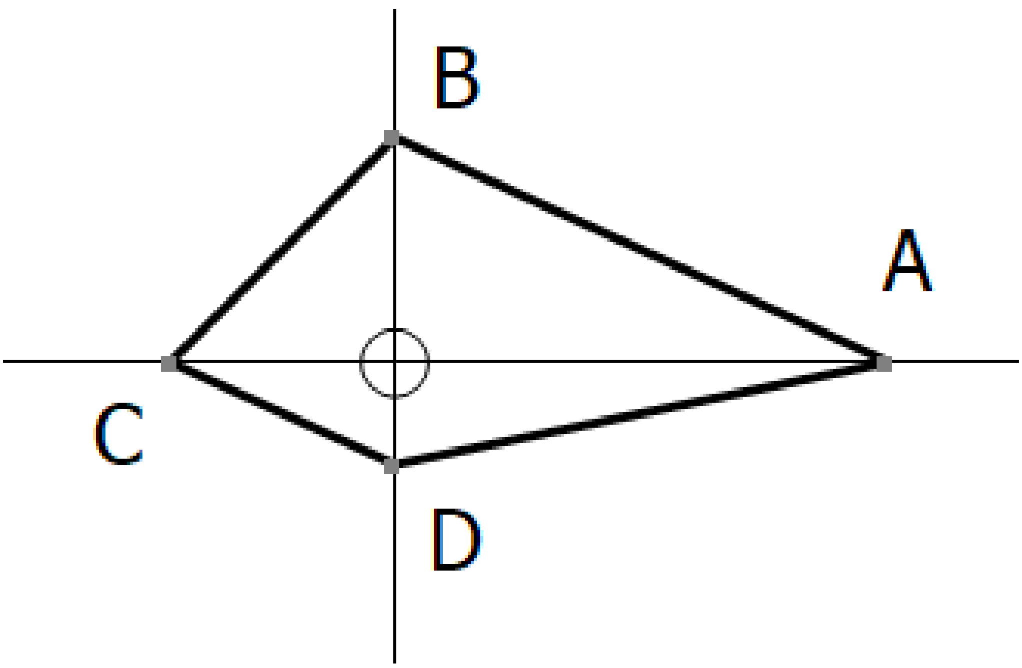
$10(a + b) + 100 = -1$, and so $a + b = -10.1$.

Thus the remainder when we divide by a + b is

$10(-10.1)^2 + 100(-10.1) + 10 = 20.1$.

So on multiplying by 10, 201 is the final answer.

4.



Gradient of AB is $-\frac{b}{a}$, the gradient of CD is $-\frac{1/a}{1/b}=-\frac{b}{a}$, thus AB and CD are parallel.

Area = $\frac{1}{2}(ab+1+1+\frac{1}{ab})=\frac{a^2b^2+2ab+1}{2ab}=\frac{(ab+1)^2}{2ab}$.

If a = 11, then area = $\frac{(11b+1)^2}{22b}=\frac{1}{22}\left(121b+22+\frac{1}{b}\right)$.

The numbers 121b and 1/b have a constant product, so 121b + 1/b is minimised when 121b = 1/b, which gives b = 1/11.

So on multiplying by 1018000 and taking the integer part, 92545 is the final answer.

5. The spreadsheet below shows you what happens;

Start	cos(left)	arctan(left)	
1	0.540302306	0.495367289	
0.495367289	0.879794176	0.721538843	
0.721538843	0.75079015	0.644006613	
0.644006613	0.799696595	0.674555912	
0.674555912	0.780984384	0.663038028	
0.663038028	0.788125922	0.667458598	
0.667458598	0.785397308	0.665773221	
0.665773221	0.786439427	0.666417432	
0.666417432	0.786041356	0.666171432	
0.666171432	0.786193403	0.666265405	
0.666265405	0.786135326	0.666229512	
0.666229512	0.786157509	0.666243222	
0.666243222	0.786149036	0.666237985	
0.666237985	0.786152272	0.666239985	
0.666239985	0.786151036	0.666239221	
0.666239221	0.786151508	0.666239513	
0.666239513	0.786151328	0.666239402	
0.666239402	0.786151397	0.666239444	
0.666239444	0.78615137	0.666239428	
0.666239428	0.786151381	0.666239434	
0.666239434	0.786151377	0.666239432	
0.666239432	0.786151378	0.666239433	
0.666239433	0.786151378	0.666239432	
0.666239432	0.786151378	0.666239433	
0.666239433	0.786151378	0.666239432	
0.666239432	0.786151378	0.666239432	
0.666239432	0.786151378	0.666239432	
0.666239432	0.786151378	0.666239432	0.119911945

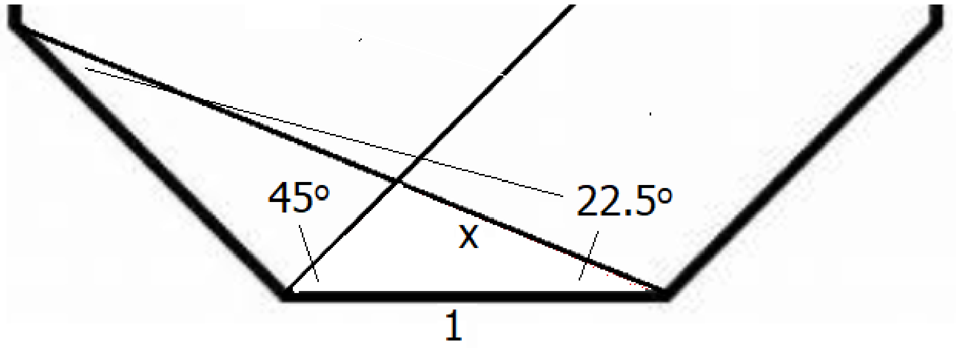
So on multiplying by 609 and taking the integer part, 73 is the final answer.

6. Overestimate = area of trapezium – area under curve =

$$\frac{(n^2+(n+1)^2)\times 1}{2}-\int_n^{n+1}x^2dx=n^2+n+\frac{1}{2}-\left[\frac{x^3}{3}\right]_n^{n+1}=n^2+n+\frac{1}{2}-\frac{(n+1)^3}{3}+\frac{n^3}{3}=\frac{1}{6}$$

So on multiplying by 1212, 73 is the final answer.

7. Let the octagon have side 1, which means the area of the octagon is $2+2\sqrt{2}$.



From the sine rule, $\dfrac{x}{\sin 45^o} = \dfrac{1}{\sin 112.5^o} \Rightarrow x = \dfrac{\sin 45^o}{\sin 112.5^o}$.

The area of the triangle is therefore (using A = $\frac{1}{2}ab\sin C$)

$$\frac{1}{2}x\sin 22.5^o = \frac{\sin 22.5^o \sin 45^o}{2\sin 112.5^o} = 0.146.....$$

Thus the percentage shaded is $\dfrac{0.146...}{2+2\sqrt{2}} \times 100\% = 3.033008....$

So on multiplying by 155 and taking the integer part, 470 is the final answer.

8. The only values that work are;

Score	1	2	3	4	5	6
Freq	2	4	5	1	3	6

So this gives us $\sum x^2 = 370.$

So 370 is the final answer.

9. Using a spreadsheet here gives;

n	(n/10)^2	floor((n/10)^2)	ceiling ((n/10)^2)	ceiling + floor	u_n+1-u_n
1	0.01	0	1	1	x
2	0.04	0	1	1	0
3	0.09	0	1	1	0
4	0.16	0	1	1	0
5	0.25	0	1	1	0
6	0.36	0	1	1	0
7	0.49	0	1	1	0
8	0.64	0	1	1	0
9	0.81	0	1	1	0
10	1	1	1	2	1
11	1.21	1	2	3	1
12	1.44	1	2	3	0
13	1.69	1	2	3	0
14	1.96	1	2	3	0
60	36	36	36	72	3
61	37.21	37	38	75	3
62	38.44	38	39	77	2
63	39.69	39	40	79	2
64	40.96	40	41	81	2
65	42.25	42	43	85	4
66	43.56	43	44	87	2
67	44.89	44	45	89	2
68	46.24	46	47	93	4
69	47.61	47	48	95	2
70	49	49	49	98	3
71	50.41	50	51	101	3
72	51.84	51	52	103	2

Thus 64 is the value required.

So on multiplying by 1013, 64832 is the final answer.

10. We know $S_\infty = \dfrac{a}{1-r}$. Thus

$$\frac{x}{1-y}=2, \frac{y}{1-x}=3 \Rightarrow \frac{x}{1-3(1-x)}=2 \Rightarrow x=\frac{4}{5}, y=\frac{3}{5}$$

Thus $xy=\frac{12}{25}$.

So on multiplying by 147300, 70704 is the final answer.

11. We have, where x, y and z are the missing digits,

652641310 = 10501 + 1010x + 3406043+10100y + 649040946+z101000.

Thus 183820=1010(100z+10y+x), and so 100z+10y+x=182

Thus z = 1, y = 8, x = 2, and the product of the six digits is 256.

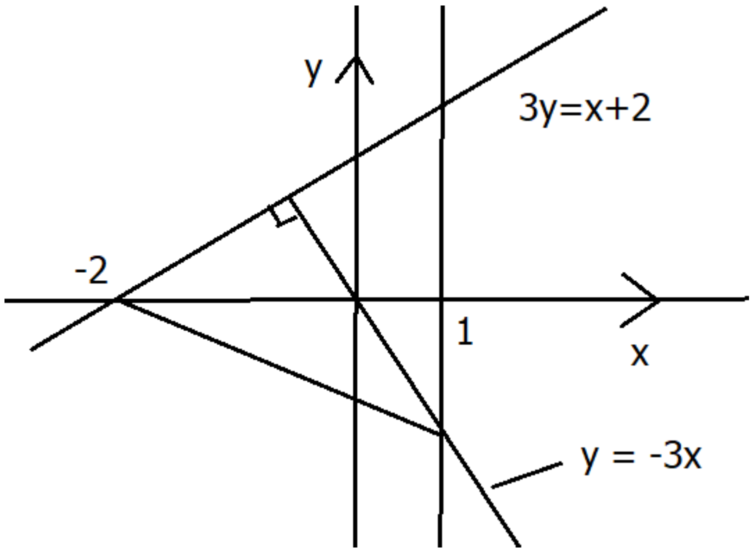
So on multiplying by 176, 45056 is the final answer.

12.

$$\frac{10!/(2!2!2!4!)}{5!/2!} = 315.$$

So on multiplying by 132.1 and taking the integer part, 41611 is the final answer.

13.



The three vertices of the triangle are (1,1), (1,-3) and (-2,0).

Thus its perimeter is $4+\sqrt{3^2+3^2}+\sqrt{3^2+1^2}=4+3\sqrt{2}+\sqrt{10}$

So on multiplying by 4069 and taking the integer part, 46406 is the final answer.

14. If we expand the bracket, we get the sum of terms of the form

$$\binom{10}{r}(6x^3)^r\left(-\frac{5}{x^2}\right)^{10-r}$$

For this to be independent of x, 3r=2(10-r), which gives r = 4.

So this term becomes

$$\binom{10}{4}(6)^4(-5)^6 = 4252500000.$$

So on dividing by200000and taking the integer part, 21262 is the final answer.

15. If y = asin(bx + c), then the period is affected by b, but not by a or c. (The period here is 360°/b.) The same is true for cos and tan (although the period for tan is 180°.)

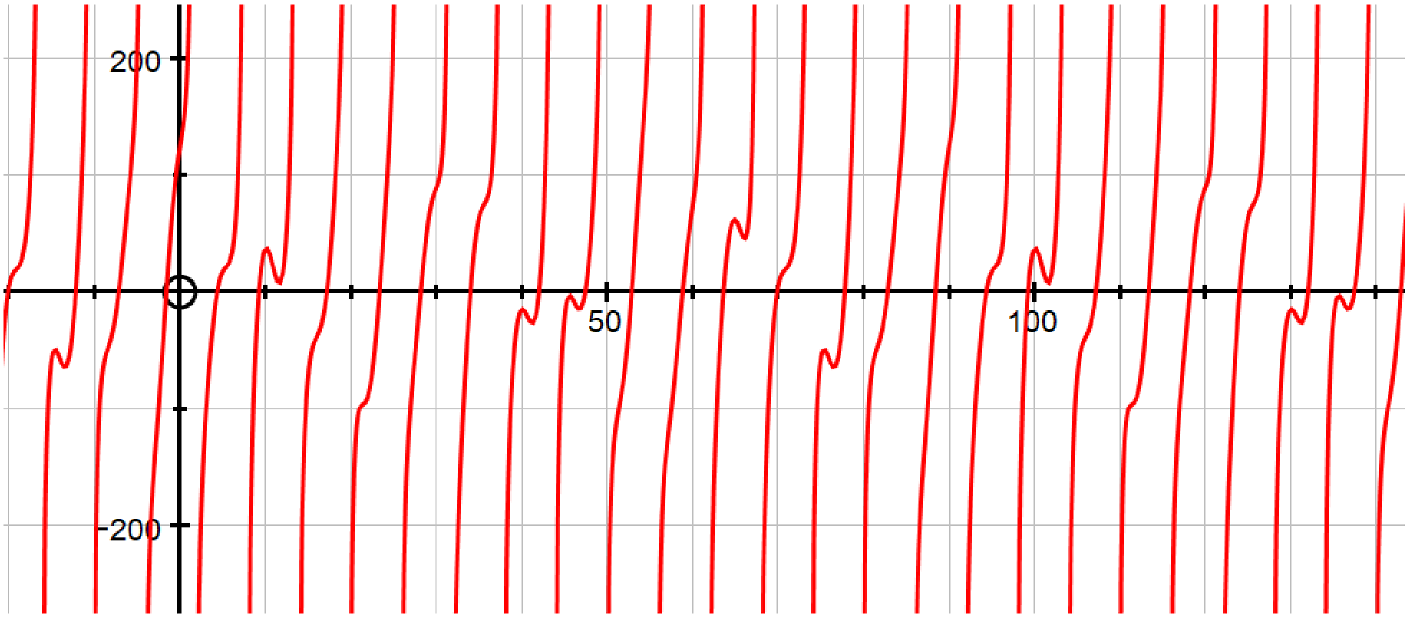
Thus the period of 30sin(40x + 72°) is 360°/40 = 9°, and

The period of 40cos(72x + 30°) is 360°/72 = 5°, and

The period of 72tan(30x + 40°) is 180°/30 = 6°.

So we need the LCM of 9, 5, and 6, which is 90. The function y has a period of 90°.

So in degrees, 90 is the final answer.



200
-200
50
100

16. We see that $46656 = 2^6 3^6$, so b and d can take the values 2, 3, 4, 6.

(a,c)	2=d	3=d	4=d	6=d
2=b	$(2,2^2 3^3)$ $(3,3^2 2^3)$ $(2^2,2^1 3^3)$ $(2^1 3^1,2^2 3^2)$ $(2^3,3^3)$ $(3^2,2^3 3)$ $(2^2 3,2^1 3^2)$	$(2^3,3^2)$	$(2^1 3^1,2^1 3^1)$	x
3=b	$(2^2, 3^3)$	$(2, 2^1 3^2)$ $(2^1 3^1,2^1 3^1)$ $(3,2^2 3)$ $(2^2,3^2)$	x	x
4=b	$(2,2^1 3^3)$ $(3,2^3 3)$ $(2^1 3^1,2^1 3^1)$		x	x
6=b	$(2,3^3)$ $(3,2^3)$	$(2,3^2)$ $(3,2^2)$	x	(2,3)

So there are 22 possibilities.

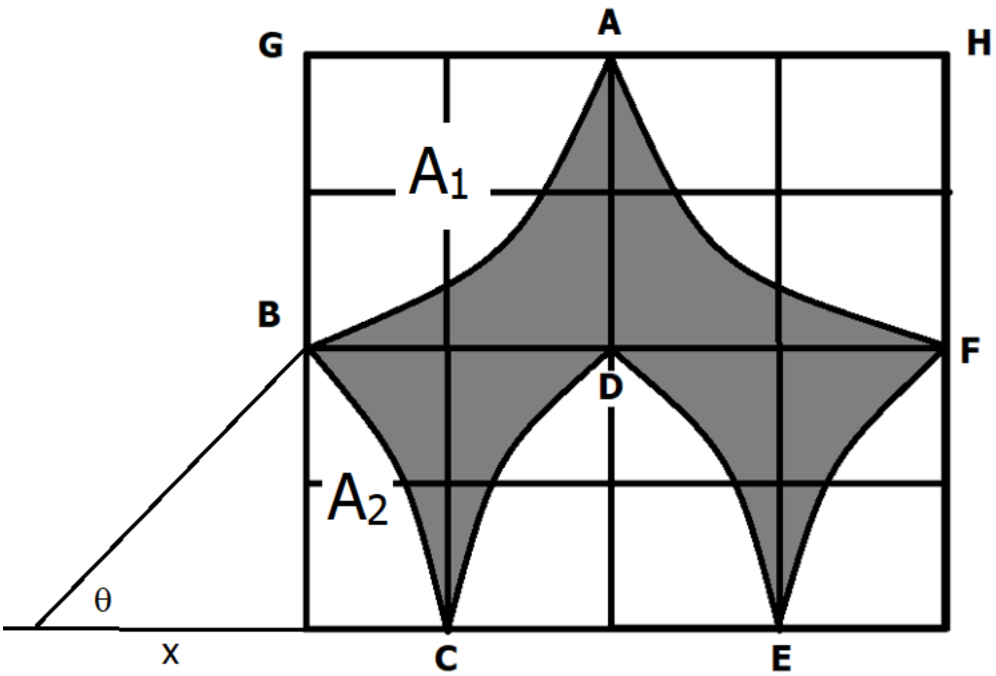
The simplest way to tackle this is probably to run a program.

1	*a*	*b*	*c*	*d*	
2	2	2	108	2	46656
3	2	3	18	3	46656
4	2	4	54	2	46656
5	2	6	3	6	46656
6	2	6	9	3	46656
7	2	6	27	2	46656
8	3	2	72	2	46656
9	3	3	12	3	46656
10	3	4	24	2	46656
11	3	6	4	3	46656
12	3	6	8	2	46656
13	4	2	54	2	46656
14	4	3	9	3	46656
15	4	3	27	2	46656
16	6	2	6	4	46656
17	6	2	36	2	46656
18	6	3	6	3	46656
19	6	4	6	2	46656
20	8	2	9	3	46656
21	8	2	27	2	46656
22	9	2	24	2	46656
23	12	2	18	2	46656

So on multiplying by 10^3, 22000 is the final answer.

17. The shaded area is $16 - 2A_1 - 4A_2$, where

$$A_1 = \frac{4\pi}{4} = \pi.$$



Let the radius of the circle containing the arc BC be r.

Then $r = x+1$, and $r^2=x^2+4$.

Thus $(1+x)^2 = x^2 + 4$ and $x = 1.5$, and $\tan\theta = 4/3$.

Now $A_2 = \frac{1}{2}\arctan(4/3)(2.5)^2 - \frac{1}{2}2(1.5) = 1.397...$

So shaded area is 4.125624469…

So on multiplying by 10^4 and taking the integer part, 41256 is the final answer.

18. $f'(x) = 3ax^2+2bx+c$, $f''(x) = 6ax+2b$, $f'''(x) = 6a$.

So $3=6a$, $a = 0.5$, and $2 = 6 + 2b$, $b = -2$, and $1 = 1.5 - 4 + c$, $c=3.5$, $d = 0$.

Thus $(c+d)^{(a+b)}=3.5^{-1.5}=0.1527207097...$

So on multiplying by 3340 and taking the integer part, 510 is the final answer.

19. 2 x Area ABM = Area AMC, so BM = 1/3, MC = 2/3.

Similarly AN = NM = y.

Cos rule in ABM; 4y² = 1+ 1/9 $-2\frac{1}{3}\cos\frac{\pi}{3}$, so $y = \frac{\sqrt{7}}{6}$.

Sin rule in ABM; $\frac{\sin(AMB)}{1} = \frac{\sqrt{3}/2}{2\sqrt{7}/6} = \frac{3\sqrt{3}}{2\sqrt{7}}$ = sin AMC.

Thus cos AMC = $\sqrt{1-\left(\frac{27}{28}\right)} = \frac{1}{\sqrt{28}}$.

Cos rule in NMC; x² = y²+4/9 - $2y\frac{2}{3}\sqrt{\frac{1}{28}}$, so x = 0.7264831573…

So on multiplying by 30809 and taking the integer part, 22382 is the final answer.

20.

$$\sum_{1}^{100}(n^2\int_{\sqrt[n]{n}}^{\sqrt[n]{n+1}} x^{n-1}dx) = \sum_{1}^{100}\left(n^2\left[\frac{x^n}{n}\right]_{\sqrt[n]{n}}^{\sqrt[n]{n+1}}\right)$$

$$= \sum_{1}^{100}\Big(n\big((n+1)-(n)\big)\Big) = \sum_{1}^{100} n = 5050.$$

So on multiplying by 10 and taking the integer part, 50500 is the final answer.

21. P’ = 2x + 2y, P = 4x, A’ = xy, A = x^2.

So we have 2x+2y +d = 4x, 4x+d = xy, xy + d = x^2.

So d = 2x-2y, yielding 4x +(2x-2y) = xy, xy + (2x-2y) = x^2.

So we have $y = \frac{6x}{x+2} \Rightarrow x\frac{6x}{x+2} + 2x - 2\frac{6x}{x+2} = x^2.$

Solving this gives x = 0, y = 0 (not allowed), x = 4,y = 4 (not allowed), and x = 2, y = 3. So our four terms are P’ = 10, P = 8, A’ = 6, A = 4.

The sum of these values is 28.

So 28 is the final answer.

22. Multiplying out, $a + \sqrt{b} = (5c - 3db) + (5d - 3c)\sqrt{b}$.

Since b is not a square, $\sqrt{b}$ is irrational, so for this equation to work,

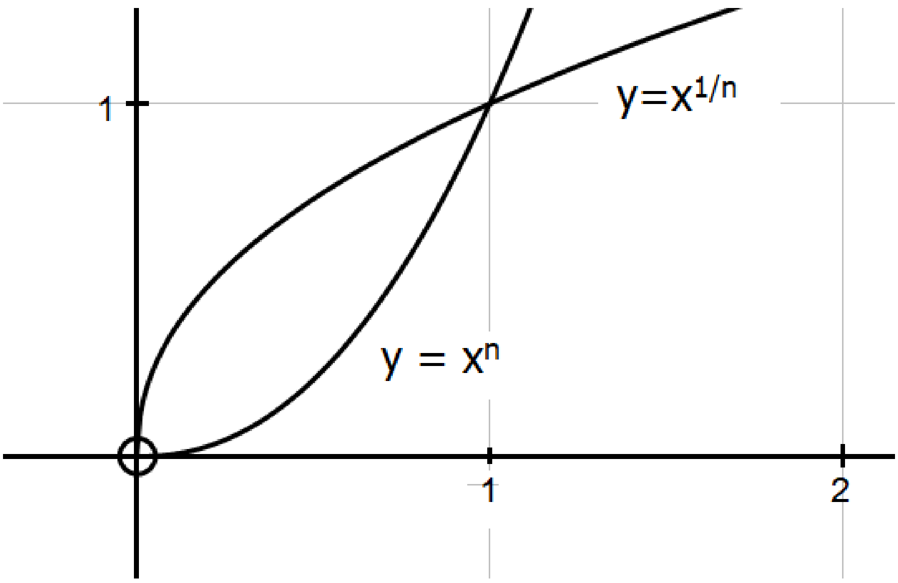
a = 5c-3db, 5d-3c = 1. If 0<c,d<7, there is only one solutions to 5d-3c=1, that is, d = 2, c = 3.

So we also need to solve a = 15 – 6b, or a + 6b = 15, and a = 3, b = 2 is the only solution in our range.

Thus a = 3, b = 2, c = 3, d = 2, and the product of these is 36.

So 36 is the final answer.

23.



Enclosed area =

$$\int_0^1 x^{1/n} - x^n dx = \left[\frac{x^{n+1/n}}{n+1/n} - \frac{x^{n+1}}{n+1} \right]_0^1$$

$$= \frac{n}{n+1} - \frac{1}{n+1} = \frac{n-1}{n+1} = \frac{1}{100} \Rightarrow n = \frac{101}{99.}$$

So on multiplying by 10^7 and taking the integer part, 10202020 is the final answer.

24. Say the three angles in degrees are a, ar and ar^2.

So a + ar + ar^2 = 180, and $a(ar)(ar^2)=20 = (ar)^3$.

Thus $ar = \sqrt[3]{20} = k \Rightarrow r = \frac{k}{a}$. So

$$a + k + \frac{k^2}{a} = 180 \Rightarrow a^2 + (k-180)a + k^2 = 0$$

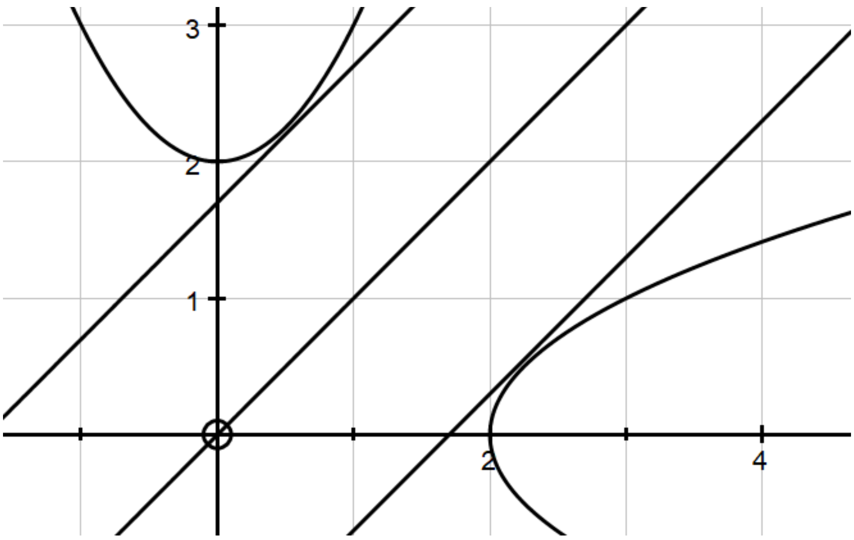
$$\Rightarrow a = \frac{180 - k \pm \sqrt{(180-k)^2 - 4k^2}}{2}$$

Substituting in for k gives a = 177.24…. or 0.0415701659… (in degrees).

We want the smaller of these.

So on multiplying by 10^9 and taking the integer part, 41570165 is the final answer.

25.



The parabola $y = x^2 + 2$ is the reflection of the parabola $x = y^2 + 2$ in $y = x$.

This means the smallest AB can be is when the gradient of the curves at both A and B are 1.

At A, $y' = 2x = 1$, so $x = 0.5$, $y = 2.25$, so B is (2.25,0.5).

Thus the distance AB is $\sqrt{2(2.25-0.5)^2} = 1.75\sqrt{2} = 2.474873734...$

So on multiplying by 10^3 and taking the integer part, 2474 is the final answer.