

# Ritangle 2018 – Final Question Answers

## **Round 1: What connects?**

a.  $(5,4), (1+\sqrt{7},1+3\sqrt{2}), (4,5), (1+\sqrt{5},1+2\sqrt{5}), (1+\sqrt{k},3)$

If we plot the first four points, we see they lie on the same circle,

The centre of the circle must lie on  $y = x$ ,

since  $(4, 5)$  and  $(5, 4)$  are on the circle. Call the centre  $(a, a)$ .

We now have  $r^2 = (a - 4)^2 + (a - 5)^2$ , and  $r^2 = (a - (1 + \sqrt{5}))^2 + (a - (1 + 2\sqrt{5}))^2$ ,

and solving these for  $a$  gives  $a = 1, r = 5$ .

$$\text{Thus } 25 = k + 4, k = 21.$$

- b. **Circle,**  $\frac{1}{4\pi}$   
**Equilateral triangle,**  $\frac{\sqrt{3}}{36}$   
**Square,**  $\frac{1}{16}$   
**Regular hexagon,**  $k$

It seems likely that area or perimeter may be involved.

Is there a constant involving the area and the perimeter

that is independent of the size of the shape?

The simplest such expression is  $\frac{\text{area}}{\text{perimeter}^2}$  which gives the values in part b.

The circle gives the largest possible value for this for all possible shapes.

The regular hexagon comes next here,

followed by the square and then the triangle.

The value of  $k$  we need is  $\frac{\sqrt{3}}{24} = 0.072168\dots$

So multiplying by  $10^4$  and taking the integer part gives 721.

$$c. \int_0^6 x + 1 \, dx \qquad \int_0^6 x^2 + x - 11 \, dx$$

$$\int_0^6 x^3 + x^2 + x - 65 \, dx \qquad \int_0^6 x^4 + x^3 + x^2 + x - k \, dx$$

The value of the first three integrals is 24. The value of k required to make the fourth integral equal to 24 is 324.2.

So multiplying by 10 gives 3242.

### **Round 2: Sequences**

*a.* 0, 0, 2, 6, 12, 21, 34, 51, *k*...

a. The sequence is given by writing down the cubes starting with 1, and then losing the units digit. Thus *k* = 72.

*b.* 2, 2, 3, 5, 11, 19, 41, *k*...

b. The numbers are all prime numbers.

The sequence given is  $p_1, p_1, p_2, p_3, p_5, p_8, p_{13} \dots$ ,

where  $p_n$  is the  $n^{\text{th}}$  prime number.

In other words, we just take the prime numbers with Fibonacci subscripts.

Thus the next in the sequence is  $p_{21} = 73$ .

c.

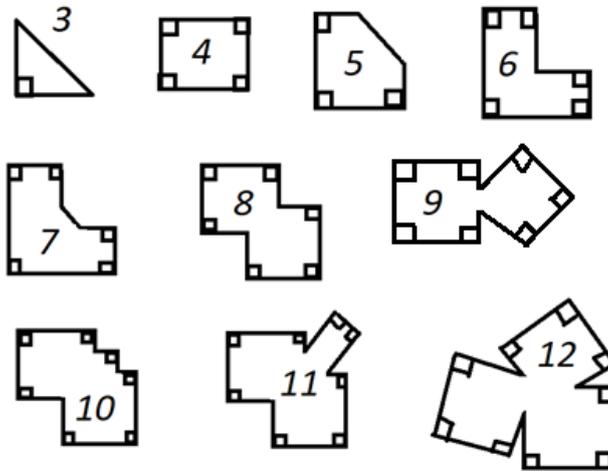
3	4	5	6	7	8	...	12
1	4	3	5	5	6	...	k

Hint for c.



The sequence is built like this;

Given a number  $n$  in the top row, what is the maximum number of right angles an  $n$ -sided shape can have?



So the value of  $k$  we need is 9.

### ***Round 3: The Wall***

a.

**3 34 36 37**

**55 78 91 100**

**101 144 210 509**

**528 610 625 729**

The four quartets are;

3, 37, 101, 509 (prime)

144, 34, 610, 55 (Fibonacci)

100, 36, 729, 625 (square)

78, 528, 91, 210 (triangle).

So these should be entered in this order;

**337101509**

**3455144610**

**36100625729**

**7891210528**

b.

**Vivid Tesla Pascal Lux**

**Mix Katal Sharp Cobra**

**Hardy Lovelace Mill Newton**

**Latex Henry Lisp Civil**

c. The four quartets are;

Vivid, Mix, Civil, Mill (words spelled with Roman numerals);

Hardy, Sharp, Lovelace, Newton (English mathematicians);

Tesla, Katal, Henry, Lux (S.I. units)

Latex, Lisp, Cobra, Pascal (computing languages)

So these should be entered in this order;

**CivilMillMixVivid**

**CobraLatexLispPascal**

**HardyLovelaceNewtonSharp**

**HenryKatalLuxTesla**



220	10648000
222	10941048
224	11239424
226	11543176
228	11852352
230	12167000
232	12487168
234	12812904
236	13144256
238	13481272
240	13824000
242	14172488
244	14526784
246	14886936
248	15252992
250	15625000
252	16003008
254	16387064
256	16777216
258	17173512
260	17576000
262	17984728
264	18399744
266	18821096

So the following needs to be entered as the unique answer; **24648866**

c) fncnsndhrnvrss

d) *hxndhnrllogfx*

e) *hhrdrfxndxhhr*

f) *nvrxndnvr*

These become;

c) functionsandtheirinverses

d) *etothexandthenaturallogofx*

e) *thethirdrootofxandxtothethree*

f) *oneoverxandoneoverx*

### **Round 5: The Crossnumber**

The grid needs to spell 'Ritangle'.

So the 9 clues need to be in the order

**19/46, 17, 65/90, 65/90, 19/46, 10, 85, 82, 36, 57, 30**

Now the final answers for questions 1-25

will fit into the diagram to form a crossnumber.

2	0	2		2	4	4	9	4	8	9	7		4	5	0	5	6		2	8	
1		4		2		1		6		2	0	1		1	0		4		2		
2	6	7		0		2		4	1	5	7	0	1	6	5		3	8		3	6
6		4	7	0		5		0		4	0		5	1	0		7	3		8	
2			9	0		6		6		5	4			1	0	2	0	2	0	2	0