HE Mathematics

Projectiles
Section 2: General equations
Exercise

Level 1

1. Find the Cartesian equation of the path of these projectiles by eliminating the parameter t:
   (i) $x = 3t,\ y = 4t^2$
   (ii) $x = 7t,\ y = 8t - 5t^2$
   (iii) $x = 5t,\ y = 3 + 2t - 4t^2$

2. **Use 10 m s^{-2} for g in this question**
   A projectile is launched from the origin at an angle of 60° to the horizontal with an initial velocity of 40 m s^{-1}.
   (i) Write down the x and y coordinates of the projectile after time t.
   (ii) Show that the equation of trajectory of the projectile is $y = x\sqrt{3} - \frac{1}{80}x^2$.
   (iii) Find y when $x = 5$.
   (iv) Find x when $y = 20$.

Level 2

3. A man throws a ball horizontally from the top of a hill 4.9 m high. He wants the ball to clear a fence 2.4 m high standing on horizontal ground and 8 m horizontally away from the point of projection. Find the minimum speed at which the ball must be thrown. If the ball is thrown with this minimum speed find how far beyond the fence that it lands.

4. Shells are fired from a gun at 210 ms^{-1}. What is the maximum range of the shells on horizontal ground? At what angle should the shells be fired if they are to hit a target 3600 m away?

5. A free kick in football is taken from point O on horizontal ground. 2 seconds later it is at a height of 2.4 m and 22 m away from where it was kicked. Find
   (i) The velocity at which it was kicked
   (ii) The maximum height that it achieved
   (iii) The distance from O at which the ball lands

6. (i) Freddie hits a cricket ball at an initial speed of 25 ms^{-1} and an angle of 50° to the horizontal. Find the greatest height reached by the ball.
   (ii) The pavilion is 50 m away from Freddie and is 10 m high. Will the ball clear the pavilion?

7. Take g to be 10 ms^{-2} for this question.
   A particle is projected at 60 ms^{-1} at an angle α such that $\tan\alpha = \frac{4}{3}$ from a point O on a horizontal plane. Find

(i) The time at which the particle is at a height of 99 m above the plane
(ii) The horizontal distances from O when it is at a height of 99 m.

Level 3

8. A particle is projected with speed u ms^{-1} on horizontal ground so that it reaches a height of more than 1.2 metres. Find the angle to the horizontal at which it must be projected if the difference in time between reaching 1.2 metres on the way up and down is 1 second and the distance apart of the points where it does so is 7 metres. (Take g = 10).

9. A particle is projected from the ground on horizontal terrain at a speed u ms^{-1} and inclination to the horizontal θ. Take the origin of coordinates as the point of projection, the x-axis as horizontal and the y-axis as vertical, both in the plane of the particle's motion.

   (i) Show that the equation of its trajectory is $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

   By regarding this as a function of x and completing the square, show that the maximum height of the particle is $\dfrac{u^2\sin^2\theta}{2g}$. At what distance horizontally from the starting point is that maximum height achieved?

   (ii) Using the identity $\dfrac{1}{\cos^2\theta} \equiv 1 + \tan^2\theta$, the equation of the trajectory can also be written as $y = x\tan\theta - \dfrac{gx^2}{2u^2}\left(1+\tan^2\theta\right)$

   Use this result to write down a quadratic equation which must be satisfied by $\tan\theta$ if the path passes through the point with coordinates $\left(x_0, y_0\right)$.

   If $x_0 \neq 0$, find conditions in terms of u and g for which

   (A) the equation has two distinct roots;
   (B) the equation has two equal roots;
   (C) the equation has no roots.

   Deduce that the point $\left(x_0, y_0\right)$ is *inaccessible* if $y_0 > \dfrac{u^2}{2g} - \dfrac{gx_0^2}{2u^2}$.

